Part 3, Appendix A: Derivation of Equations for Case 3

A.1 Dimensions *X* and *Y* when the soil pressure distribution forms a rectangle and a triangle

When the soil pressure distribution under the footing forms a combination of a rectangle and a triangle as shown in the Figure A-1, let *X* represent the length of the rectangular portion and *Y* the length of the triangular portion.

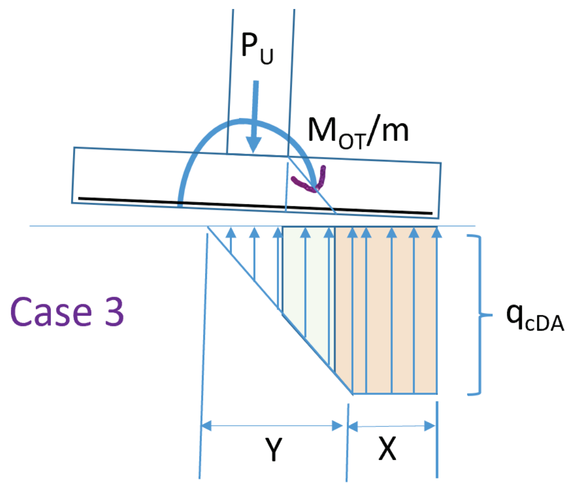

Figure A-1 Soil pressure distribution under the footing forms a rectangle and a triangle.

Taking moments about the center line of the footing:

$$M_{UD} = \ q_{cDA}B_fX\left(\frac{L_f}{2}-\frac{X}{2}\right)+\frac{q_{cDA}B_fY}{2}\left(\frac{L_f}{2}-X-\frac{1}{3}Y\right);$$

or

$$\frac{2M_{UD}}{q_{cDA}B_f} = \ L_fX - X^2 + \frac{L_fY}{2} - XY - \frac{Y^2}{3};$$

or

$$\frac{2M_{UD}}{q_{cDA}B_f} = \ L_f\left(X+\frac{Y}{2}\right) - X^2 - XY - \frac{Y^2}{3} \qquad Eq.A-1$$

Summing the axial forces on the footing

$$\left(X + \frac{Y}{2}\right) = \frac{P_U}{q_{cDA}B_f} \qquad Eq. A - 2$$

Substituting equation A-2 in equation A-1.

$$\frac{2M_{UD}}{q_{cDA}B_f} = \frac{P_U L_f}{q_{cDA}B_f} - X^2 - XY - \frac{Y^2}{3}; \qquad Eq. A - 3$$

From equation A-2

$$X = \frac{P_U}{q_{cDA}B_f} - \frac{Y}{2};$$

Therefore,

$$X^2 = \left(\frac{P_U}{q_{cDA}B_f}\right)^2 - \frac{P_U}{q_{cDA}B_f}Y + \frac{Y^2}{4};$$

And

$$XY = \frac{P_U Y}{q_{cDA}B_f} - \frac{1}{2}Y^2;$$

Substituting for X² and XY in Eq, A3 and expanding we get:

$$\frac{2M_{UD}}{q_{cDA}B_f} - \frac{P_U L_f}{q_{cDA}B_f} = -\left\{\left(\frac{P_U}{q_{cDA}B_f}\right)^2 + \frac{Y^2}{12}\right\}$$

or

$$\frac{Y^2}{12} = \frac{P_U L_f}{q_{cDA}B_f} - \frac{2M_{UD}}{q_{cDA}B_f} - \left(\frac{P_U}{q_{cDA}B_f}\right)^2$$

or

$$Y = \sqrt{12\left\{\frac{P_U L_f}{q_{cDA}B_f} - \frac{2M_{UD}}{q_{cDA}B_f} - \left(\frac{P_U}{q_{cDA}B_f}\right)^2\right\}}$$

Let

$$P' = \frac{P_U}{q_{cDA}B_f}$$

$$M' = \frac{M_{UD}}{q_{cDA}B_f}$$

Then

$$Y = \sqrt{12\left\{P'L_f - 2M' \ - P'^2\right\}}$$

and

$$X = P' - \frac{1}{2}Y$$